Now that you understand what it means to divide with fractions, take a look at this problem.

Micah is running a 6-mile race. There are water stops every $\frac{1}{2}$ mile, including at the 6-mile finish line. How many water stops will there be?

Explore It

Use the math you already know to solve this problem.

- You want to find how many groups of $\frac{1}{2}$ there are in 6. Write a division expression that represents the problem. _________

- You can draw a model to help you solve the problem. Label the number line below from 0 to 6.

- Draw points on the number line to show the location of all the water stops.

- How many $\frac{1}{2}$s are in 1? _________

- How many $\frac{1}{2}$s are in 6? _________

- Explain how you can use the number line to find the number of water stops there will be in all. ____________________________

______________________________
Dividing fractions can be modeled in many different ways.

Another way to show \(6 \div \frac{1}{2}\) is by drawing a model. You can draw 6 rectangles to represent the 6 miles.

Draw lines to show the halves.

You can also use common denominators to divide 6 by \(\frac{1}{2}\). Consider these questions:

How many wholes are in 12?
How many groups of ten are in 12 tens?
How many groups of one hundred are in 12 hundreds?
How many groups of one tenth are in 12 tenths?

Reflect

1. How can you find \(3 \div \frac{1}{6}\)?
Read the problem below. Then explore different ways to understand dividing a fraction to solve word problems.

Piper used \( \frac{1}{5} \) meter of ribbon to create a border around a triangle. If the sides of the triangle are all the same length, how much ribbon did she use for each side?

**Picture It**

You can draw a picture to help understand the problem.

Draw a 1-meter length of ribbon, then draw and label \( \frac{1}{5} \)-meter lengths.

Divide each \( \frac{1}{5} \)-meter length into 3 equal parts.

**Model It**

You can use a model to help understand the problem.

Draw and shade \( \frac{1}{5} \) of a rectangle.

Divide the rectangle into 3 equal parts.
Part 2: Guided Instruction

Connect It

Now you will solve the problem from the previous page using your understanding of fractions and the models.

2 Look at Picture It on the previous page. What information in the problem does the first picture show?

____________________________________________________________________________________

3 Why does the second picture show each bar divided into 3 parts?

____________________________________________________________________________________

4 When you divide $\frac{1}{5}$ meter into 3 equal parts, how long is each part? _________ meter
   How do you know? ____________________________________________________________________
   ________________________________________________________________________________

5 How much ribbon did Piper use for each side of the triangle? _________

6 What division expression represents this problem? _________

7 What is $\frac{1}{5} \div 3$? _________

8 Describe how Model It on the previous page shows dividing a fraction by a whole number. ____________________________________________________________________________
   ________________________________________________________________________________

Try It

Use what you just learned about dividing fractions to solve this problem. Show your work on a separate sheet of paper.

9 Tate has $\frac{1}{4}$ of a pizza. He wants to share the pizza equally with a friend. How much of the original whole pizza will each of them get? Draw a model and write an equation for your answer.

____________________________________________________________________________________
Read the problem below. Then explore different ways to understand dividing a whole number by a fraction.

Alex makes 2 pounds of bread dough. He needs to separate the dough into \(\frac{1}{4}\)-pound loaves for the oven. How many loaf pans will Alex need to bake all the loaves at the same time?

**Picture It**

You can draw a model to help understand the problem.

Draw 2 rectangles to represent the 2 pounds of bread dough.

Draw lines to show dividing each pound into fourths.

**Model It**

You can use a number line to help understand the problem.

Draw a number line and label it to show the 2 pounds of bread dough.

Mark the number line to show dividing into fourths.
Connect It

Now you will solve the problem from the previous page using the models and equations.

10 Write a division expression that represents the problem. _______________________

11 How many fourths are in one whole? _______
   How many fourths are in two wholes? _______

12 Use this information to solve the problem.
   Using words:
   There are _______ fourths in 2.

   Using numbers:
   \[ 2 \div \frac{1}{4} = _______ \]

13 What multiplication sentence could you write to check your answer? ________________

14 Describe a strategy you can use to divide a whole number by a unit fraction.

Try It

Use what you just learned about dividing whole numbers by fractions to solve this problem. Show your work on a separate sheet of paper.

15 Stacy has 4 sheets of paper to make cards. Each card requires \( \frac{1}{2} \) sheet of paper. How many cards can Stacy make? Choose a strategy to solve the problem. Then explain why you chose that strategy.
Study the model below. Then solve problems 16–18.

Sierra has 3 empty pages for photos in an album. Each photo uses \( \frac{1}{6} \) of a page. How many photos can Sierra put on the empty pages if she only places photos on 1 side of each page?

**Look at how you could show your work using rectangles.**

```
  +---+---+---+---+---+
  |   |   |   |   |   |
  +---+---+---+---+---+
  |   |   |   |   |   |
  +---+---+---+---+---+
  |   |   |   |   |   |
  +---+---+---+---+---+
```

6 photos will fit on one page, so 18 photos will fit on 3 pages.

**Solution:** 18 photos

16 Corrine picked \( \frac{1}{4} \) gallon of blackberries. She separated the berries equally into 4 containers. What fraction of a gallon is in each container?

**Show your work.**
17 Cooper’s USB drive is \( \frac{1}{2} \) full with 5 video files. Each video file is the same size. What fraction of the USB drive does 1 video file use?

Show your work.

**Solution:** 

18 Devonte is creating note cards to help him study for a history test. He fills 2 full sheets of paper and uses \( \frac{1}{8} \) of a sheet for each major event he needs to study. Which expression shows the number of major events Devonte needs to study for the test? Circle the letter of the correct answer.

A  \( 2 \times \frac{1}{8} \)

B  \( 2 \div \frac{1}{8} \)

C  \( \frac{1}{8} \times 2 \)

D  \( \frac{1}{8} \div 2 \)

Barry chose D as the correct answer. How did he get that answer?

---

Pair/Share

How could I represent this problem using an equation?

How can you check your answer?

Is this problem like any problem I’ve seen before?

Does Barry’s answer make sense?
Solve the problems. Mark your answers to problems 1–3 on the Answer Form to the right. Be sure to show your work.

1. Which equation could the shaded section of the model represent?

   A \( 4 \div \frac{1}{3} = \frac{1}{12} \)
   B \( \frac{1}{3} \div 4 = \frac{1}{12} \)
   C \( 3 \times \frac{1}{12} = 4 \)
   D \( 4 \times \frac{1}{3} = \frac{1}{12} \)

2. Elise picks 6 pounds of apples. She uses \(\frac{1}{2}\) pound of apples to make 1 container of applesauce. How many containers of applesauce can Elise make?

   A 12 containers
   B \(6 \frac{1}{2}\) containers
   C \(5 \frac{1}{2}\) containers
   D 3 containers
3 Ramon and 2 of his friends will run in a relay race. The length of the race is $\frac{1}{3}$ mile. If each person runs the same distance, how far will Ramon run?

A 1 mile  
B $\frac{2}{3}$ mile  
C $\frac{1}{6}$ mile  
D $\frac{1}{9}$ mile

4 Marina has a pattern to make bows that requires $\frac{1}{4}$ yard of ribbon for each bow.

**Part A**

Fill in the table to show how many bows she can make from a given length of ribbon.

<table>
<thead>
<tr>
<th>Yards of Ribbon</th>
<th>Number of Bows</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

**Part B**

Use words or an equation to describe a rule to find the number of bows Marina can make if you know how many yards of ribbon she has.

**Part C**

Use your rule to find how many bows Marina can make if she has 18 yards of ribbon.

**Answer** __________ bows

**Self Check**

*Go back and see what you can check off on the Self Check on page 85.*
Lesson 18 (Student Book pages 150–159)

Divide Unit Fractions in Word Problems

LESSON OBJECTIVES

• Represent and solve real-world problems involving division of unit fractions by whole numbers using visual fraction models and equations.
• Represent and solve real-world problems involving division of whole numbers by unit fractions using visual fraction models and equations.

PREREQUISITE SKILLS

• Divide whole numbers.
• Multiply whole numbers by unit fractions.
• Understand that multiplication and division are inverse operations.
• Use a visual fraction model to find the quotient of a unit fraction divided by a whole number.
• Use a visual fraction model to find the quotient of a whole number divided by a unit fraction.

VOCABULARY

There is no new vocabulary. Review the following key terms.

divide: to separate a whole into equal groups
multiply: to find the total number of items in equal-sized groups inverse operations

THE LEARNING PROGRESSION

This lesson builds on the previous lesson, which showed students how to use visual fraction models and reasoning about the inverse relationship between multiplication and division to divide a unit fraction by a whole number and a whole number by a unit fraction. In this lesson, students solve real-world problems involving division of whole numbers and unit fractions using equations as well as visual fraction models.

In Grade 6, students build on these foundational skills when they solve real-world problems involving division of fractions by fractions using visual fraction models and equations.

CCLS Focus

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share \( \frac{1}{2} \) lb of chocolate equally? How many \( \frac{1}{3} \)-cup servings are in \( 2 \) cups of raisins?

ADDITIONAL STANDARDS: 5.NF.7.a, 5.NF.7.b (see page A32 for full text)

STANDARDS FOR MATHEMATICAL PRACTICE: SMP 1–8 (see page A9 for full text)
Lesson 18
Part 1: Introduction

Divide Unit Fractions in Word Problems

Now that you understand what it means to divide with fractions, take a look at this problem.

Micah is running a 6-mile race. There are water stops every $\frac{1}{2}$ mile, including at the 6-mile finish line. How many water stops will there be?

Use the math you already know to solve this problem.

• You want to find how many groups of $\frac{1}{2}$ there are in 6. Write a division expression that represents the problem: $\frac{6}{\frac{1}{2}}$.

• You can draw a model to help you solve the problem. Label the number line below from 0 to 6.

• Draw points on the number line to show the location of all the water stops.

• How many $\frac{1}{2}$s are in 1?

• How many $\frac{1}{2}$s are in 6? [12]

• Explain how you can use the number line to find the number of water stops there will be in all. Possible answers: If you mark all of the water stops, you can just count.

Possible answer: if you mark all of the water stops, you can just count.

Mathematical Discourse

• How did you figure out the number of water stops in 1 mile?

There are 2 halves in 1 whole. So, when I divide 1 by $\frac{1}{2}$, I divide 1 whole into 2 parts. Each half-mile has a water stop, so there are 2 stops in 1 whole mile.

• How does it help you to know the number of water stops in 1 mile?

Think of each mile as a “group.” If I know the number in each group, I can multiply by the number of groups to find how many stops there are in all. So, 6 miles—or groups—with 2 halves (water stops) in each group equals 12 half-miles, or 12 water stops.
Part 1: Introduction

AT A GLANCE

Students explore different ways to model division of fractions. They draw a visual model and find a common denominator to use in a mathematical model.

STEP BY STEP

• Read Find Out More as a class.

• Discuss how the visual model represents halves and wholes.

• Discuss the questions about units. Relate the idea of a unit to the meaning of a denominator: the denominator tells what size the parts (or units) are.

• Ask, How does finding a common denominator help me divide 6 by \( \frac{1}{2} \)? [When we write 6 as a fraction with the denominator 2, the numerator tells how many halves are in 6.]

• Students complete Reflect individually. Then have students share their responses and discuss.

SMP Tip: When working with fractions, students must pay careful attention to the units being considered (SMP 6). As students work these problems, continue to ask, What are the units?

ELL Support

Review with students the mathematical meaning of a unit. A unit is the type of measurement being used, or how we explain what one is, for a given problem. With the water stops problem, the unit was halves. This page talks about ones, tens, hundreds, tenths, and halves as units. When measuring, units might be cups, pounds, inches, etc. (Students are also familiar with the idea of a “unit of study.” This is a different usage of the word, meaning a related knowledge that is studied together for a period of time.)

Hands-On Activity

Use fraction strips to divide.

Materials: fraction strips

• Have students model 6 wholes.
• Have students use fraction strips to model each whole as 2 halves.

Real-World Connection

Encourage students to think about everyday places or situations where people might need to divide unit fractions.

Example: cooking recipes

Bring in copies of recipes that include several ingredients with amounts expressed as a unit fraction (e.g., \( \frac{1}{4} \) cup, \( \frac{1}{2} \) tsp, etc.). Tell students that you have a certain whole-number amount of an ingredient. For that ingredient, have them use division to calculate how many batches of the recipe you could make.
Students read a word problem involving dividing a fraction by a whole number and explore ways to illustrate the problem and create a visual model.

**STEP BY STEP**

- Read the problem at the top of the page as a class.
- Discuss the meaning of the problem.
- Guide students through the process of drawing the picture shown in Picture It. Point out how the picture illustrates each part of the word problem.
- Read Model It with students. Discuss how the model is like other area models students have drawn and how it is alike and different from the picture.
- Ask, **What does the dark-shaded part represent?** [The length of ribbon used for 1 side of the triangle.]

**SMP Tip:** Help students develop a consistent approach to making sense of word problems (SMP 1) by modeling and discussing good practice. Model reading the problem and discussing its meaning in your own words. Model making an estimate of the solution and creating an illustration or visual model. After solving a problem, model how to check an answer for reasonableness.

### Mathematical Discourse

- **Why do you think the picture and model show 1 whole meter when the problem is about \( \frac{1}{5} \) meter?**
  
  Showing the whole meter helps us understand what \( \frac{1}{5} \) meter is. This helps clarify the meaning of the problem and keeps the rest of the model accurate so it can help us find a correct solution.

- **How are the picture and the model alike? How are they different?**
  
  Both show the whole meter and \( \frac{1}{5} \) meter. Both show the 3 sides. The picture looks more like an actual ribbon. The model uses area and shading to show the three sides and the length of one side.

### Hands-On Activity

**Materials:** yarn, scissors, glue, paper, ruler

- Give each student a piece of paper with an equilateral triangle with a side length of 6 inches drawn on it.
- Give each student a piece of yarn \( \frac{1}{2} \) yard long and tell them that is is \( \frac{1}{2} \) yard long.
- Have students glue the piece of yarn around the border of the triangle.
- Have students measure the length of each side of the triangle with the yarn glued to it. [6 inches]
- Have students convert the side length of the triangle to yards \( \left( \frac{1}{6} \text{ yard} \right) \), and explain to them that this is the quotient when dividing \( \frac{1}{2} \) by 3.
Students revisit the problem on page 152 and use the picture and model to solve the problem. Then, they apply their knowledge to model and solve another problem.

**STEP BY STEP**

- Work through Connect It with students. Have them look at the picture and model on page 152 while they answer questions about the problem.
- Have students complete problems 2–5 individually.
- Discuss answers to problems 4 and 5 as a class. Point out that it is necessary to understand the whole meter in order to answer problem 4.
- Have students write their answers to problems 6 and 7. Then discuss problems 6–8 as a class. Have students point out the features of the model that show dividing a fraction by a whole number.
- Have students complete Try It on their own. Then organize them into small groups to discuss their models and solutions. To help students understand this problem, you may want to model this problem using a circular pizza cut out of paper.

**SMP Tip:** As students develop models (including equations) for problems (SMP 4), ask them to explain how each feature of their model represents the quantities and the situation in the original problem. Students’ models are only as useful as their connection to the problem being solved.

### TRY IT SOLUTIONS

9. **Solution:** \( \frac{1}{4} \div 2 = \frac{1}{8} \) pizza; Students may draw a model showing a circle divided into fourths, with \( \frac{1}{4} \) shaded. Then, further divide the circle into eighths and shade half of the fourth in a darker color.

**ERROR ALERT:** Students who wrote \( \frac{1}{2} \) may have multiplied \( \frac{1}{4} \) by 2 instead of dividing, or they may have answered the question, “How much of the piece will Taylor give to his friend?”
AT A GLANCE

Students read a word problem involving dividing a whole number by a fraction and explore ways to illustrate the problem and create a visual model.

STEP BY STEP

- Read the problem at the top of the page as a class.
- Discuss the meaning of the problem.
- Guide students through the process of drawing the picture shown in Picture It. Ask, What does each large rectangle represent? [a pound of dough] What does each part of a rectangle represent? [\(\frac{1}{4}\) pound of dough]
- Discuss the number line in Model It. Ask how the number line represents the problem situation. Have students compare the number line to the picture.

Hands-On Activity

Act out the problem situation.

Materials: modeling compound

- Have students use two equal-sized lumps of modeling compound to represent 2 pounds of bread dough.
- Have students separate each clump into fourths (4 equal-sized pieces).
- Ask, In real life, do you think a baker would carefully measure the weight of the dough for each loaf? Why or why not? Small bakeries or people who bake their own bread might not worry about getting the weight exactly accurate, but large commercial bakeries are likely to be very meticulous about the weight of the dough for each loaf in order to control quality and cost.

Mathematical Discourse

- Why do you think this is a division problem?
  It is division because Alex is separating the dough into equal-sized groups. The size of the groups is known: \(\frac{1}{4}\) pound each. The quotient will tell how many groups he made.
- How are the picture and model alike? How are they different?
  They both show 2 pounds; the picture uses 2 rectangles and the number line goes from 0 to 2. They both show the pounds divided into fourths. The picture does not use numbers.
Students revisit the problem on page 154 and use the models shown to solve the problem by dividing $2 \div \frac{1}{4}$. They describe a general strategy for dividing a whole number by a unit fraction and apply the strategy to solve a problem.

**STEP BY STEP**

- Explain that Connect It uses the problem and visual models from page 154.
- Have students write their answer to problem 10 and then check their answers as a group.
- After students complete problems 11 and 12, discuss as a class. Have students explain their reasoning.
- After students complete problems 13 and 14, discuss as a class.
- Have students complete Try It on their own. Then have volunteers explain their strategies and show their models and solutions to the class. Make a chart listing the kinds of models students made and their reasons for choosing that kind of model.

**TRY IT SOLUTIONS**

15 **Solution:** 8; Students may draw a number line from 0 to 4 to represent 4 sheets of paper. Then mark halves along the number line. Finally, show each “jump” of $\frac{1}{2}$ from 0 to 4 and count them. Students may also write the division equation $4 \div \frac{1}{2} = 8$ and check their work using multiplication. $8 \times \frac{1}{2} = 4$

**ERROR ALERT:** Students who wrote 2 may have multiplied instead of dividing.
Students study a model for solving a word problem that involves unit-fraction division. Then, they solve several word problems and share their solution strategies with a partner.

**STEP BY STEP**

- Ask students to solve the problems individually as you circulate and provide support. Guide students as needed to develop meaningful visual models that correctly represent the problem situations.
- When students have completed each problem, have them Pair/Share to discuss their solutions with a partner or in a group.

**SOLUTIONS**

**Example (Ex)** A model is shown as one way to solve the problem. Students could also solve the problem by solving the equation \( 3 \div \frac{1}{6} \).

**Solution:** \( \frac{1}{6} \) gallon; Students could solve the problem by drawing a model that shows \( \frac{1}{6} \) gallon and is divided into 4 equal parts.

**Solution:** \( \frac{1}{10} \) of the drive; Students could solve the problem by writing and solving the equation \( \frac{1}{2} \div 5 \).

**Solution:** B; The starting amount, 2 pages, is divided into groups that are \( \frac{1}{8} \) page in size. Explain to students why the other two answer choices are not correct:

- A is not correct because it represents 2 groups that are each \( \frac{1}{8} \) of a page.
- C is not correct because it represents \( \frac{1}{8} \) of 2 pages.
Part 5: Common Core Practice

Lesson 18

At a Glance

Students divide unit fractions to solve word problems that might appear on a mathematics test.

Step by Step

• First, tell students that they will divide unit fractions to solve word problems. Then have students read the directions and answer the questions independently. Remind students to fill in the correct answer choices on the Answer Form.

• After students have completed the Common Core Practice problems, review and discuss correct answers. Have students record the number of correct answers in the box provided.

Solutions

1 Solution: B; \( \frac{1}{3} \) of the whole is shaded. The shaded third is divided into 4 equal parts, and one of the parts is shaded in a darker color.

2 Solution: A; Each pound has two halves so it can make 2 containers of applesauce. Elise has 6 pounds, so that is 12 halves and 12 containers.

3 Solution: D; There are 3 runners, so divide \( \frac{1}{3} \) mile into 3 equal portions. Each portion is \( \frac{1}{9} \) mile.

4 Part A Solution: 4, 8, 12, 16; Each bow needs \( \frac{1}{4} \) yard, and there are 4 fourths in 1 whole. So, for each yard Marina can make 4 bows.

Part B Solution: for example, number of yards \( \div \frac{1}{4} \). Part C Solution: for example, \( 18 \div \frac{1}{4} = 72 \) bows.
Assessment and Remediation

- Ask students to draw a diagram, write an equation, and solve the following problem: A container has \( \frac{1}{5} \) of a gallon of juice. If 5 friends share the juice, how much does each friend get? \( \frac{1}{25} \) gallon
- For students who are struggling, use the chart below to guide remediation.
- After providing remediation, check students’ understanding. Ask students to draw a diagram, write an equation, and solve the following problem: Raquel picked 8 cups of strawberries. She makes \( \frac{1}{3} \) cup servings for a picnic. How many servings can she make from 8 cups? [24 servings]
- If a student is still having difficulty, use Ready Instruction, Level 5, Lesson 17.

<table>
<thead>
<tr>
<th>If the error is . . .</th>
<th>Students may . . .</th>
<th>To remediate . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>have thought that the answer is the number of parts in the whole after dividing.</td>
<td>Have students draw an area model or a number line and identify ( \frac{1}{5} ). Then, have students divide the fifth into 5 equal-sized pieces. Point out that each friend gets one of these pieces, which is ( \frac{1}{5} ) of ( \frac{1}{5} ) or ( \frac{1}{5} \times \frac{1}{5} ).</td>
</tr>
<tr>
<td>1</td>
<td>have multiplied instead of dividing.</td>
<td>Write ( \frac{1}{5} \times 5 )” and “( \frac{1}{5} \div 5 )” and discuss what each expression means in the context of the problem. ( \frac{1}{5} \times 5 ) could mean “( \frac{1}{5} ) of 5 friends” or “each of 5 friends has ( \frac{1}{5} ) gallon.” These do not describe the problem situation. ( \frac{1}{5} \div 5 ) means ( \frac{1}{5} ) gallon divided (or shared) among 5 friends,” which describes the problem situation exactly.</td>
</tr>
</tbody>
</table>

Hands-On Activity

Use fraction circles to divide by a fraction.

Materials: fraction circles

Use 6 whole fraction circles to represent. Set up the problem: “A painter can paint \( \frac{1}{3} \) of a room in an hour. Suppose that 6 rooms need painting and it all has to be done in an hour. How many painters are needed?” Have students represent the 6 whole rooms with 6 whole circles. Ask, How much of a room can 1 painter paint? \( \frac{1}{3} \) of the room How many painters are needed to paint 1 room? [3 painters] Have students cover each whole circle with thirds to represent the painters needed to paint each room. Ask, How many thirds are there in 6 rooms? [18, so you need 18 painters] Write \( 6 \div \frac{1}{3} = 18 \). Repeat the activity with other fractions if time allows.

Challenge Activity

Materials: index cards

Write division expressions on several index cards. Write some involving dividing a unit fraction by a whole number and some involving dividing a whole number by a unit fraction. Students pick a card and write their names on the back. Students write a word problem to match their expressions and write their names on the back. Have students swap word problems and write an equation to solve each problem. Have students compare their equations to the original expression and discuss whether the problems and equations are correct. If errors are found, have students make corrections. Challenge students to come up with several different scenarios for each expression.