Lesson 17 Part 1: Introduction

Solve Problems with Inequalities

CCLS
7.EE.3
7.EE.4.b

You've learned how to solve two-step equations. Take a look at this problem.

Mr. Thomas brings $100 to a fundraiser. He wants to leave the event with at least $50 in his pocket. Guests at the fundraiser buy raffle tickets for several different prizes. Each raffle ticket costs $2.50. How many raffle tickets can Mr. Thomas buy and still leave with at least $50 in his pocket?

Explore It

Use math you already know to solve the problem.

- How much money does Mr. Thomas have at the start of the fundraiser? _____
- Let t = the number of tickets bought. Write an expression to show how much it costs to by t tickets. _____
- Use the expression above to write a different expression that shows how much money Mr. Thomas would have left after buying t tickets. _____
- Suppose Mr. Thomas buys 25 tickets. How much money would he have left? Is this at least $50? Show your work.

  _____

- Suppose Mr. Thomas buys 20 tickets. How much money would he have left? Is this at least $50?

  _____

- What is the greatest number of tickets Mr. Thomas can buy and still have $50 left? Explain. _____

  _____

- Could Mr. Thomas buy fewer than 20 tickets? Explain.

  _____

  _____

- Fill in the blank. Mr. Thomas could buy any number of tickets that is _____ or less.

Find Out More

You can also solve the problem on the previous page by writing and solving an inequality.

(starting amount) − (ticket price) • (number of tickets) is greater than/equal to (amount left)

$100 \quad - \quad 2.5 \quad \cdot \quad t \quad \geq \quad 50$

Solving a two-step inequality is similar to solving a two-step equation. But, when you multiply or divide each side of the inequality by a negative number, you reverse the inequality symbol.

$$100 - 2.5t \geq 50$$

$$100 - 100 - 2.5t \geq 50 - 100$$

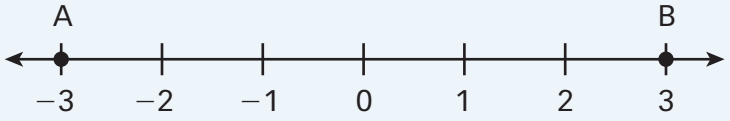
$$-2.5t \geq -50$$

$$\frac{-3t}{-2.5} \leq \frac{-50}{-2.5} \quad \text{(Reverse the symbol.)}$$

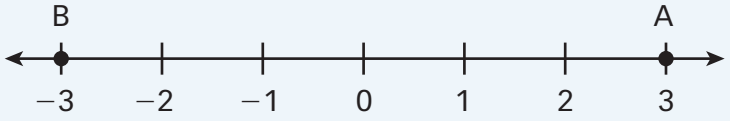
$$t \leq 20$$

He can buy 20 or fewer tickets.

Let's use integer inequalities to examine why the symbol is reversed.
A is −3 and B is 3, so, A < B. On the number line, A is to the left of 0 and B is to the right.



Now divide both A and B by −1 and compare the values. A becomes 3 and B becomes −3. Now A > B and A is to the right of 0 and B is to the left.



After dividing by a negative number, the quotient is on the opposite side of 0, which means the symbols are reversed.

Reflect

1 What happens to the inequality if you multiply both numbers by a negative number? Explain, and give an example.

Read the problem below. Then explore different ways to solve a two-step inequality.

Chang has $60 at most to spend on socks and sneakers. He finds a pair of sneakers that he likes for $36. If socks are $3 per pair, how many pairs could Chang buy?

Model It

You can write and solve an inequality to understand the problem.

The price of the sneakers and socks combined must be $60 or less.

price of sneakers $+$ $3 \cdot p$ pairs socks must be $<$ or $=$ \$ Chang has

$36 + 3p \leq 60$

$36 - 36 + 3p \leq 60 - 36$

$3p \leq 24$

$\frac{3p}{3} \leq \frac{24}{3}$

$p \leq 8$

Model It

You can graph the solution set on a number line.

The inequality $p \leq 8$ means that the solution set is any number 8 or less. In this situation, fractions do not satisfy the solution. In other words, the solution set includes only whole numbers. You can't buy a fraction of a pair of socks, or a negative pair of socks.

Connect It

Now you will look at the solution and graph to analyze and interpret the inequality.

2 What does the inequality statement mean?

3 List the steps for solving this two-step inequality, using the terms constant and coefficient.

4 Explain why the numbers between each whole number are not a part of the solution set.

5 According to the graph, what is the complete solution set for the inequality?

Why are −1 and −2 not in the solution set? _____

6 How do you graph the solution set of an inequality when that set includes only whole numbers? _____

Try It

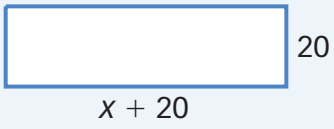
Use what you just learned about inequalities to solve these problems.

7 Students in the garden club are planting a spring flower garden in the town square. They have already spent $80 of the $200 budget. Write and solve an inequality to show how many $30 packs of bulbs they can buy. _____

8 Draw a number line and graph the solution set.

Read the problem below. Then explore different ways to solve a multi-step inequality.

Mrs. Sanchez is building a laundry room in the basement of the apartment building she owns. Given the layout of the basement, she wants the width of the room to be 20 feet and the length to be longer than the width. If she wants the area of the room to be more than 500 square feet, what could be the length? Look at the diagram Mrs. Sanchez drew.



Model It

You can write and solve an inequality to solve the problem.

The product of the width and length must be greater than 500 square feet.

width	•	length	more than	area
↓		↓	↓	↓
20	•	$(20 + x)$	$>$	500

$$20(20 + x) > 500$$

$$400 + 20x > 500$$

$$400 - 400 + 20x > 500 - 400$$

$$20x > 100$$

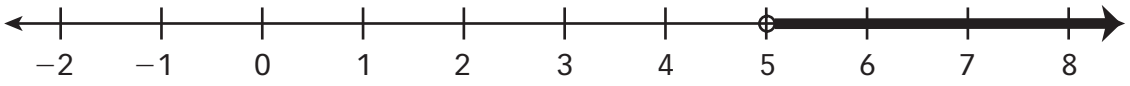
$$\frac{20x}{20} > \frac{100}{20}$$

$$x > 5$$

Model It

You can graph the solution set on a number line.

In this problem situation, the numbers in the solution can be whole numbers, decimals, or fractions. You can measure to a fraction of a foot or inch and that measurement could be used to solve the problem. So, the graph shows a solid line.



Connect It

Now you will look at the solution and graph to analyze and interpret the inequality.

9 Describe in words what the inequality statement means. _____

10 What do you do with any value for x in the solution set to find the length? Explain.

11 Look at the graph of the solution set. Why is there an open circle on the 5? Why is there a solid line instead of just circles on the whole numbers?

12 The number line ends at 10 and has an arrow to show that numbers beyond 10 are also in the solution set. Given the problem situation, do you think the solution set can extend forever, or will there be a limit? Explain. _____

13 In words, describe what the solution set is and what it means in context of the problem.

Try It

Use what you just learned about inequalities to solve this problem. Show your work on a separate sheet of paper.

14 Solve $8(12 - \frac{1}{4}x) \geq 82$ and graph the solution set on a number line.

Study the model below. Then solve problems 15–17.

Student Model

At the beginning of baseball season, Coach Thorne takes inventory of the team equipment to see what he needs. He counts 24 baseballs, but he needs to start off the season with at least 100 balls. The balls that he uses are sold in packages of 4. How many packages could the coach buy?

Look at how you could show your work by solving an inequality.

$$24 + 4x \geq 100$$
$$24 - 24 + 4x \geq 100 - 24$$
$$4x \geq 76$$
$$\frac{4x}{4} \geq \frac{76}{4}$$
$$x \geq 19$$

Solution: **Coach Thorne could buy 19 or more packages.**

The inequality shows that the 24 balls the coach has plus x packages of 4 balls each must be greater than or equal to 100.

Pair/Share

How would the inequality change if the problem said "more than 100 balls?"

15 Market and More is having a cereal sale. Every box of cereal is $0.60 off the regular price. Jane has $10 and she wants to buy 4 boxes of cereal. She uses the inequality below to determine the regular prices of boxes of cereal that she can afford. Solve the inequality and explain what the solution means.

$4(r - 0.6) \leq 10$, where r = regular price

Show your work.

Four boxes times the sale price of the box has to be ten dollars or less.

Pair/Share

Try some different prices in the solution set and discuss the results.

Solution: _____

16 Solve the inequality and graph the solution set on a number line.

$200 - 4.5x \leq 20$

Show your work.

Solution: _____

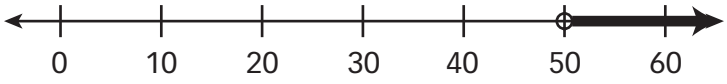
What happens when you multiply or divide both sides of an inequality by a negative number?

Pair/Share

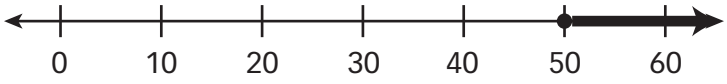
Talk about situations that this inequality might represent.

17 Greg calculated that he had to drive at least 50 miles per hour on the highway to get to his destination in the time that he has. Which number line shows the solution set to this inequality?

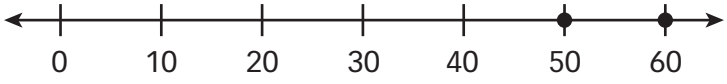
A



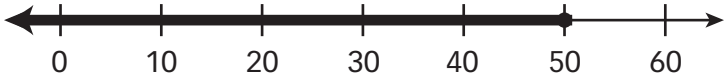
B



C



D



Jess chose **C** as the correct answer. How did she get that answer?

_____

_____

_____

_____

Think about what "at least" means in the context of the problem.

Pair/Share

Discuss what the graph in **C** means.

Answer Form

1 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ

Number Correct ___/4

Solve the problems. Mark your answers to problems 1–4 on the Answer Form to the right. Be sure to show your work.

1 What is the solution to the inequality $35 - 2x > 10$?

A $x < -12.5$

B $x > -12.5$

C $x < 12.5$

D $x > 12.5$

2 The town recreation center is sponsoring a charity drive. The goal is to raise at least $2,500. So far, $2,200 has been collected. If there are 5 days left in the charity drive, how much money on average should they aim to collect each day?

A $x \geq 500$

B $x > 440$

C $x > 60$

D $x \geq 60$

3 You need to reverse the symbol to solving which inequality?

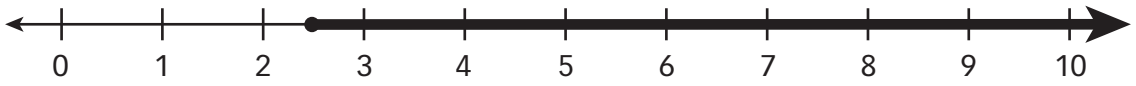
A $4(5 - y) \geq 80$

B $2(y - 3) \geq 8$

C $3y - 4 < 11$

D $-\frac{3}{4} + 6y > \frac{1}{4}$

4 The number line shows the solution set to which inequality?



A $12 + 4x > 22$

B $3x + 2 \geq 17$

C $12 + 4x \geq 22$

D $4x - 12 \leq 22$

5 The dance committee has a budget of $125 to decorate the gym for the spring dance. They have already spent $65. Some members want to buy helium balloons that cost $0.80 each. Write and solve an inequality to show the number of balloons that the dance committee could buy.

Show your work.

Answer The committee can buy _____ balloons.

6 Graph the solution set for the inequality in problem 5. Describe what the solution means and how the number of balloons bought will affect the amount in the budget.

Show your work.

Answer _____

✓ Self Check *Go back and see what you can check off on the Self Check on page 125.*

Lesson 17 (Student Book pages 156–165)

Solve Problems with Inequalities

LESSON OBJECTIVES

- Write and solve real-life inequalities that lead to the form $px + q < r$ or $px + q > r$, where p, q, and r are integers, fractions, or decimals.
- Graph and interpret the solution set of an inequality.

PREREQUISITE SKILLS

- Compute with rational numbers to solve equations.
- Write, solve, and graph inequalities.

VOCABULARY

There is no new vocabulary.

THE LEARNING PROGRESSION

In Grade 6, students studied inequalities with infinitely many solutions and represented the solutions on a number line. In this lesson, students expand upon what they learned in Grade 6 about inequalities. They solve real-world problems using inequalities and graph solutions on a number line.

Understanding inequalities is helpful in everyday life—for example, deciding how much to spend on goods and still having money left over.

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	Prerequisite Skills	*7.EE.4.b 7.EE.3*
Ready Lessons	✓	✓
Tools for Instruction	✓	✓
Interactive Tutorials	✓	✓

CCLS Focus

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations to solve problems by reasoning about the quantities.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

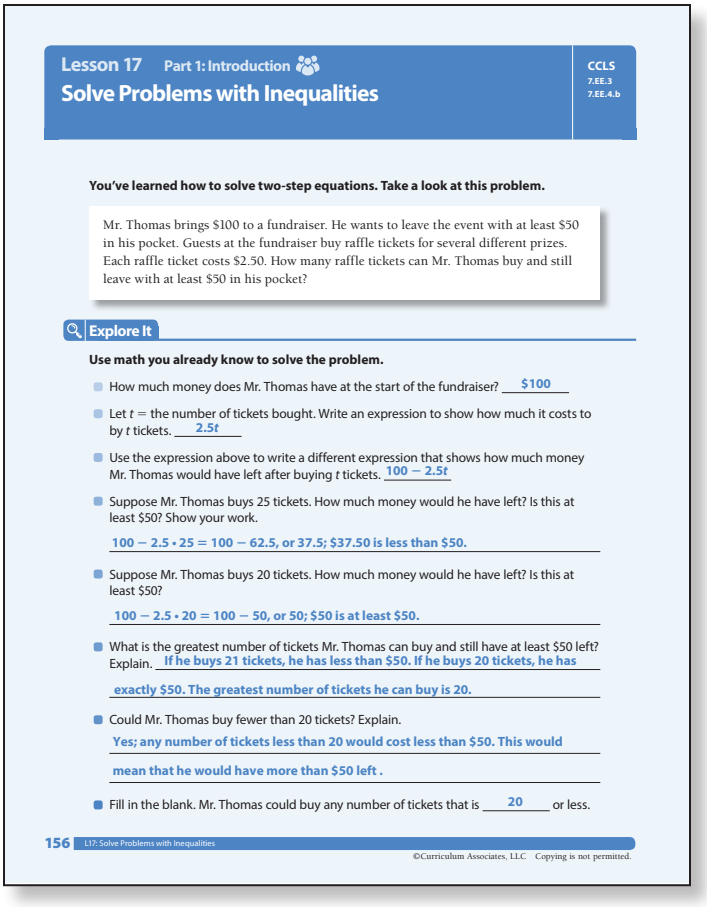
STANDARDS FOR MATHEMATICAL PRACTICE: ***SMP 1, 2, 4, 6, 7*** *(see page A9 for full text)*

AT A GLANCE

Students read a word problem that could be solved using an inequality. They then answer a series of questions to solve the problem using reasoning and their experience with equations.

STEP BY STEP

- Tell students that this page models how to solve a word problem involving an inequality.
- Have students read the problem at the top of the page.
- Work through Explore It as a class.
- Have students explain what is meant by *at least* in this problem. [greater than or equal to; The amount left over must be $50 or more.]
- Demonstrate to students that the problem is the same as finding how many raffle tickets Mr. Thomas can buy for less than $50.
- As you work through Explore It, ask students to share their responses to the bulleted items. Display their expressions and equations and discuss each step.
- Ask student pairs or groups to work on the last three bulleted items and then explain their answers for the class.



Lesson 17 Part 1: Introduction
Solve Problems with Inequalities
CCLS 7.EE.3 7.EE.4.b

You've learned how to solve two-step equations. Take a look at this problem.

Mr. Thomas brings $100 to a fundraiser. He wants to leave the event with at least $50 in his pocket. Guests at the fundraiser buy raffle tickets for several different prizes. Each raffle ticket costs $2.50. How many raffle tickets can Mr. Thomas buy and still leave with at least $50 in his pocket?

Explore It

Use math you already know to solve the problem.

- How much money does Mr. Thomas have at the start of the fundraiser? $100
- Let t = the number of tickets bought. Write an expression to show how much it costs to by t tickets. $2.5t$
- Use the expression above to write a different expression that shows how much money Mr. Thomas would have left after buying t tickets. $100 - 2.5t$
- Suppose Mr. Thomas buys 25 tickets. How much money would he have left? Is this at least $50? Show your work.
  $100 - 2.5 \cdot 25 = 100 - 62.5$, or 37.5; $37.50 is less than $50.
- Suppose Mr. Thomas buys 20 tickets. How much money would he have left? Is this at least $50?
  $100 - 2.5 \cdot 20 = 100 - 50$, or 50; $50 is at least $50.
- What is the greatest number of tickets Mr. Thomas can buy and still have at least $50 left? Explain. If he buys 21 tickets, he has less than $50. If he buys 20 tickets, he has exactly $50. The greatest number of tickets he can buy is 20.
- Could Mr. Thomas buy fewer than 20 tickets? Explain.
  Yes; any number of tickets less than 20 would cost less than $50. This would mean that he would have more than $50 left .
- Fill in the blank. Mr. Thomas could buy any number of tickets that is 20 or less.

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Hands-On Activity

Model the problem.

Materials: paper, pencils, scissors

- Divide students into small groups.
- Have each group cut out 23 small rectangles and 4 small circles.
- Label 19 five-dollar bills, 4 one-dollar bills, and 4 quarters.
- Have each group pretend to purchase raffle tickets starting with 15 tickets and keep track of the change for each purchase.

Mathematical Discourse

- *What phrases do you know that can be used to describe inequalities?*

  Students may answer with examples such as *less than*, *greater than*, *at least*, *at most*, *no less than*, and *no greater than*.

- *Suppose the situation is changed so that Mr. Thomas leaves with exactly $50. How would you change the solution?*

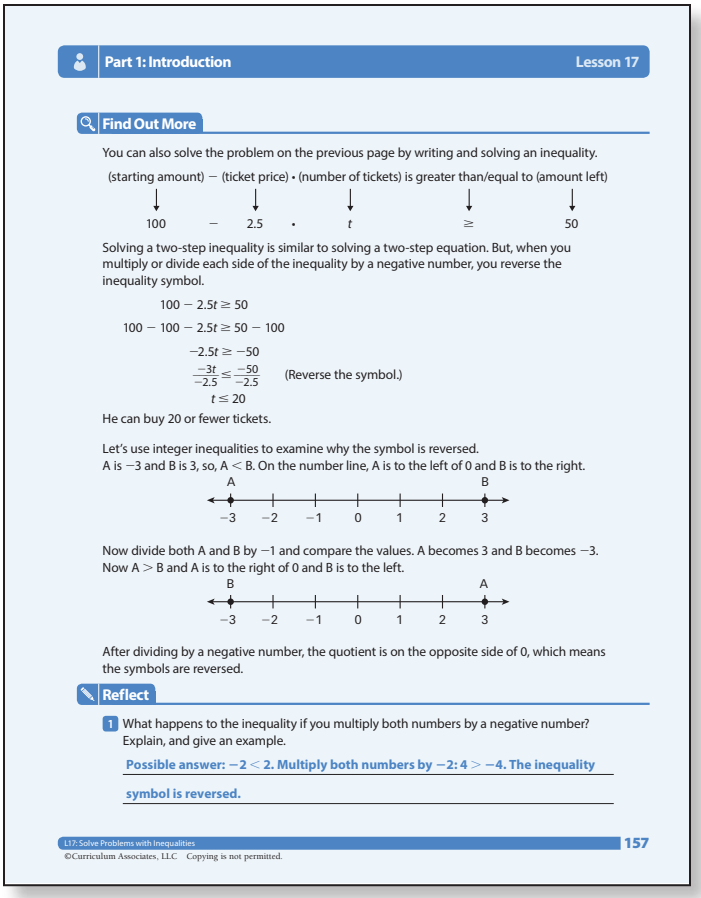
  Listen for responses that show the solution would end with Mr. Thomas buying exactly 20 tickets—no more, no less. Some students may suggest setting up and solving an equation.

AT A GLANCE

Students write an inequality to model the problem on the page 156 and then solve the inequality.

STEP BY STEP

- Read Find Out More as a class.
- Point out to students that if the problem asked for the number of tickets Mr. Thomas would buy to leave with exactly $50, they would write the equation $100 - 2.5t = 50$.
- Ask students to explain which steps are similar to solving equations.
- As you move into the second half of Find Out More, provide additional number pairs, (e.g., -3 and 6, or 4 and $-\frac{1}{2}$).
- Ask students to plot the pairs you provide on a number line, multiply or divide each number in the pair by -1, and then plot the products or quotients.
- Have students do Reflect and share their examples.



Part 1: Introduction Lesson 17

Find Out More

You can also solve the problem on the previous page by writing and solving an inequality.

(starting amount) − (ticket price) • (number of tickets) is greater than/equal to (amount left)

100 − 2.5 • t ≥ 50

Solving a two-step inequality is similar to solving a two-step equation. But, when you multiply or divide each side of the inequality by a negative number, you reverse the inequality symbol.

$100 - 2.5t \geq 50$

$100 - 100 - 2.5t \geq 50 - 100$

$-2.5t \geq -50$

$\frac{-3t}{-2.5} \leq \frac{-50}{-2.5}$ (Reverse the symbol.)

$t \leq 20$

He can buy 20 or fewer tickets.

Let's use integer inequalities to examine why the symbol is reversed.
A is −3 and B is 3, so, A < B. On the number line, A is to the left of 0 and B is to the right.

A B
−3 −2 −1 0 1 2 3

Now divide both A and B by −1 and compare the values. A becomes 3 and B becomes −3. Now A > B and A is to the right of 0 and B is to the left.

B A
−3 −2 −1 0 1 2 3

After dividing by a negative number, the quotient is on the opposite side of 0, which means the symbols are reversed.

Reflect

1 What happens to the inequality if you multiply both numbers by a negative number? Explain, and give an example.

Possible answer: −2 < 2. Multiply both numbers by −2: 4 > −4. The inequality symbol is reversed.

L17: Solve Problems with Inequalities 157
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Visual Model

Use algebra tiles to model an inequality.

Materials: negative and positive x algebra tiles

- Use algebra tiles to show $-3x \leq -6$ is equivalent to $3x \geq 6$.
- Place 3 negative x tiles on the left side of the inequality and 6 negative number tiles on the right side.
- Place 3 positive x tiles on both sides. The x tiles cancel on the left side.
- Place 6 positive number tiles on both sides. The number tiles cancel on the right side.
- This leaves 6 positive number tiles on the left side and 3 positive x tiles on the right side. The inequality is now $6 \leq 3x$.
- Discuss that $6 \leq 3x$ is equivalent to $3x \geq 6$, so $3x \geq 6$ is equivalent to $-3x \leq -6$.

Real-World Connection

Offer another situation in which it is helpful to figure the greatest or least amount needed.

Examples: the least possible grade you can get on a test without lowering your semester grade; the most each of 5 packages can weigh without going over the post-office limit.

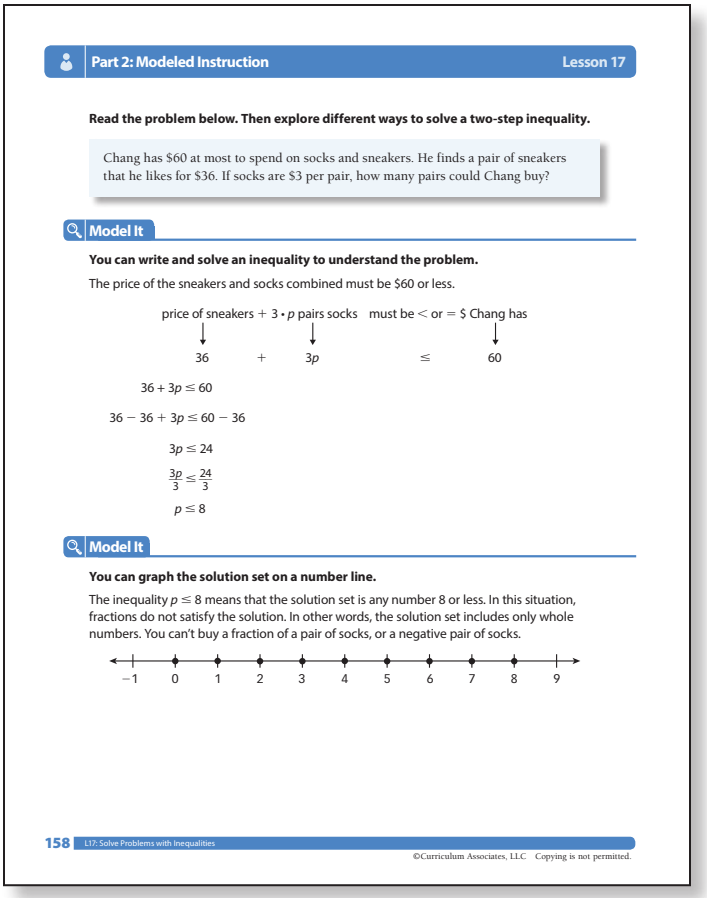
Have students offer other examples. Keep a list, and ask students to add to it during the unit.

AT A GLANCE

Students read a word problem that leads to solving an inequality. They then graph the solution set on a number line.

STEP BY STEP

- Read the problem at the top of the page as a class.
- Have students identify the given information and say in their own words what the problem is asking for.
- Have students identify the appropriate inequality. Discuss why the inequality is *less than or equal to* and not just *less than*. [He can spend $60 or less.]
- Have students examine the first Model It. Ask them to describe the steps used to solve the inequality.
- Read the second Model It. Have students examine the number line. Point out that the closed circles represent all the numbers of socks Chang can buy.
- Ask, *Could the number 9 have a closed circle? Why?* [No; 9 pairs is more than 8 pairs.]
- *Could the number* $\frac{1}{2}$ *have a closed circle? Why?* [No; you can't buy $\frac{1}{2}$ pair of socks.]
- *The number −3 is less than 8, so the inequality is true for −3. Why doesn't −3 have a closed circle?* [You can't buy a negative number of socks.]



Part 2: Modeled Instruction Lesson 17

Read the problem below. Then explore different ways to solve a two-step inequality.

Chang has $60 at most to spend on socks and sneakers. He finds a pair of sneakers that he likes for $36. If socks are $3 per pair, how many pairs could Chang buy?

Model It

You can write and solve an inequality to understand the problem.

The price of the sneakers and socks combined must be $60 or less.

price of sneakers + 3 • *p* pairs socks must be < or = $ Chang has

36 + 3*p* ≤ 60

36 − 36 + 3*p* ≤ 60 − 36

3*p* ≤ 24

$\frac{3p}{3} \leq \frac{24}{3}$

p ≤ 8

Model It

You can graph the solution set on a number line.

The inequality *p* ≤ 8 means that the solution set is any number 8 or less. In this situation, fractions do not satisfy the solution. In other words, the solution set includes only whole numbers. You can't buy a fraction of a pair of socks, or a negative pair of socks.

−1 0 1 2 3 4 5 6 7 8 9

158 L17: Solve Problems with Inequalities

Visual Model

Make a table to solve the problem.

- Construct a table on the board with four columns. Label the first column for *p*; the second column for 3*p*; the third column for 36 + 3*p* ≤ 60, *p* ≤ 8; and the fourth column for the inequality that will compare values in the third column with 60.
- Have volunteers suggest values for *p* and come up to the board and complete the table for each value.
- Discuss how to determine the correct inequality symbol.

Mathematical Discourse

- *What is another way you could solve the problem?*

  Students may describe a guess-and-check method.
- *In this problem, the answers are whole numbers. What situations may lead to answers with fractional numbers?*

  Students may give examples involving time or distance.

AT A GLANCE

Students revisit the problem on page 158 and connect the graph of the solution to the inequality that represents the problem.

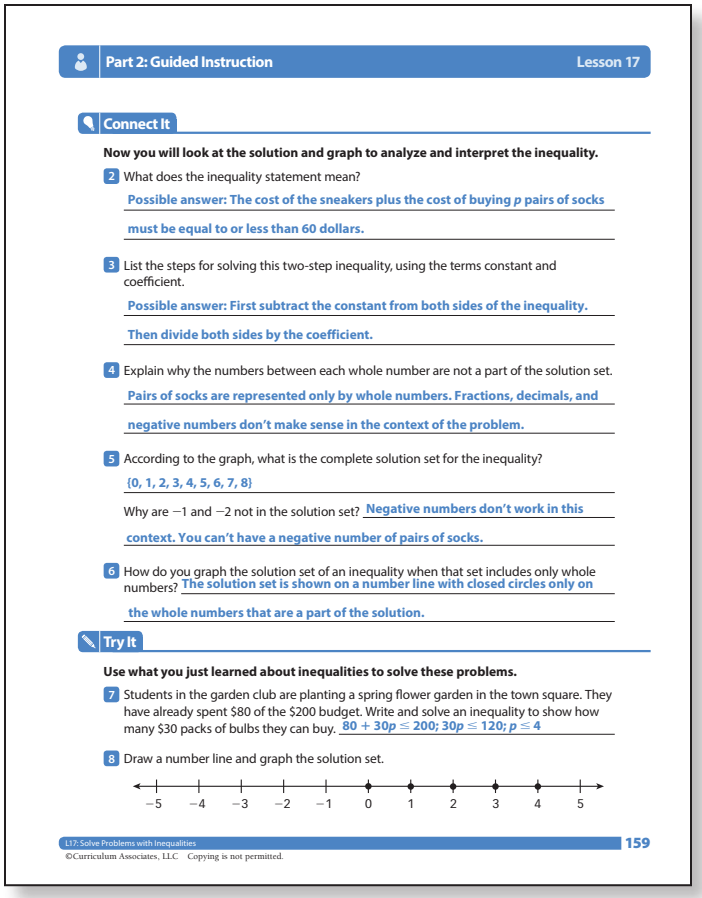
STEP BY STEP

- Read Connect It as a class. Be sure to point out that the questions refer to the problem on page 158.
- Discuss what inequality phrases can also be used to mean *at most*. [*not more than*, *less than or equal to*]
- Ask a student to demonstrate the steps for solving the inequality (i.e., problem 3). Then ask another student to solve $36 + 3p = 60$. Have the class describe similarities and differences in solving inequalities and equations.
- Emphasize the importance of context in finding a reasonable solution. There can only be a whole number of pairs of socks.
- Ask, *If the price of the sneakers changed from $36 to $35, would this change the answer? Why or why not?* [No: $1 less for the price of the sneakers would not be enough for Chang to buy another pair of socks.]
- Have students complete Try It. Ask them to Pair/Share before discussing solutions as a class.

ELL Support

Display the inequality $36 + 3p \leq 60$. Ask students to identify which numbers are constants [36, 60] and which number is a coefficient [3]. Write other inequalities for them to identify constants and coefficients. Share some ways to help remember the words. *Constant* means "not changing." The value of a *constant* does not change. The prefix *co-* means "with" or "together." People who are cooperating are working together. A *coefficient* works with a variable.

SMP Tip: Discuss with the students the importance of recognizing a situation involving comparison and representing it symbolically with the appropriate inequality symbol (*SMP 1*).



Part 2: Guided Instruction Lesson 17

Connect It

Now you will look at the solution and graph to analyze and interpret the inequality.

2 What does the inequality statement mean?

Possible answer: The cost of the sneakers plus the cost of buying *p* pairs of socks must be equal to or less than 60 dollars.

3 List the steps for solving this two-step inequality, using the terms constant and coefficient.

Possible answer: First subtract the constant from both sides of the inequality. Then divide both sides by the coefficient.

4 Explain why the numbers between each whole number are not a part of the solution set.

Pairs of socks are represented only by whole numbers. Fractions, decimals, and negative numbers don't make sense in the context of the problem.

5 According to the graph, what is the complete solution set for the inequality?

{0, 1, 2, 3, 4, 5, 6, 7, 8}

Why are −1 and −2 not in the solution set? Negative numbers don't work in this context. You can't have a negative number of pairs of socks.

6 How do you graph the solution set of an inequality when that set includes only whole numbers? The solution set is shown on a number line with closed circles only on the whole numbers that are a part of the solution.

Try It

Use what you just learned about inequalities to solve these problems.

7 Students in the garden club are planting a spring flower garden in the town square. They have already spent $80 of the $200 budget. Write and solve an inequality to show how many $30 packs of bulbs they can buy. $80 + 30p \leq 200$; $30p \leq 120$; $p \leq 4$

8 Draw a number line and graph the solution set.

−5 −4 −3 −2 −1 0 1 2 3 4 5

L17: Solve Problems with Inequalities 159

TRY IT SOLUTIONS

7 *Solution:* $80 + 30p \leq 200$; $30p \leq 120$; $p \leq 4$; Students may write an inequality to describe the situation, then solve by subtracting the constant term and dividing by the coefficient.

8 *Solution*: See answer on student page above; Students may use the inequality $p \leq 4$ to determine that the solutions are 0, 1, 2, 3, and 4, and then graph the numbers on the number line.

ERROR ALERT: Students who graphed all numbers less than or equal to 4 did not consider that only whole number packs of bulbs can be bought.

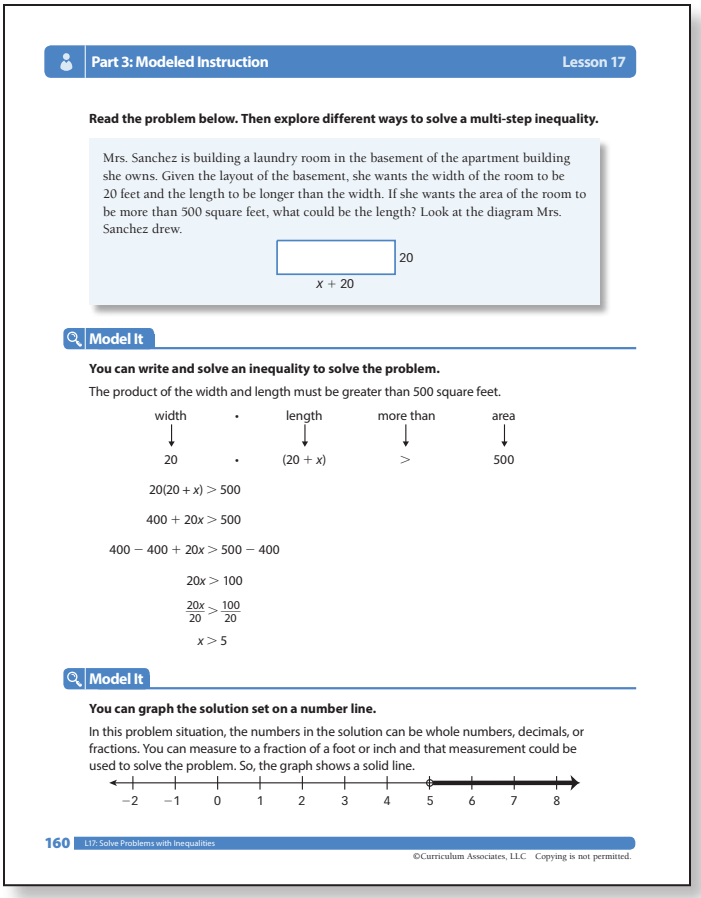
AT A GLANCE

Students read a word problem that leads to solving an inequality and graphing the solution set on a number line.

STEP BY STEP

- Read the problem at the top of the page as a class.
- Call on a student to identify the given information and state what the problem is asking for. Ask others to restate the problem in their own words, explaining that situations with inequalities can be more difficult to understand and represent than situations with equations.
- Have students look at the diagram of the basement. Ask, *What is the formula for the area of a rectangle?* [area = length × width]
- Have students examine the first Model It.
- Ask students to identify the inequality symbol. [more than, or greater than]
- Discuss why the inequality is strictly greater than. [The length can never be equal to the width; it must always be longer than the width.]
- Point out to students that the distributive property was used to get the inequality $400 + 20x > 500$.
- Read the second Model It. Ask students to compare this graph with the graph for the socks-and-sneakers situation on pages 158 and 159. [In their comparisons of the graphs, students may talk about the solutions to the socks-and-sneakers situation represented as closed points on whole numbers, while the solutions to the laundry-room situation are represented by the arrow extending rightward from the open circle over 5.]

SMP Tip: Students decontextualize the problem situation to represent it with an inequality (*SMP 2*).


Part 3: Modeled Instruction Lesson 17

Read the problem below. Then explore different ways to solve a multi-step inequality.

Mrs. Sanchez is building a laundry room in the basement of the apartment building she owns. Given the layout of the basement, she wants the width of the room to be 20 feet and the length to be longer than the width. If she wants the area of the room to be more than 500 square feet, what could be the length? Look at the diagram Mrs. Sanchez drew.

Model It

You can write and solve an inequality to solve the problem.

The product of the width and length must be greater than 500 square feet.

width · length more than area

20 · (20 + x) > 500

$20(20 + x) > 500$

$400 + 20x > 500$

$400 - 400 + 20x > 500 - 400$

$20x > 100$

$\frac{20x}{20} > \frac{100}{20}$

$x > 5$

Model It

You can graph the solution set on a number line.

In this problem situation, the numbers in the solution can be whole numbers, decimals, or fractions. You can measure to a fraction of a foot or inch and that measurement could be used to solve the problem. So, the graph shows a solid line.

−2 −1 0 1 2 3 4 5 6 7 8

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Mathematical Discourse

- *How is solving an inequality similar to solving an equation?*

  Listen for evidence that students recognize that, as with equations, they must perform the same operations on both sides of an inequality in order to isolate a variable.

- *How is solving an inequality different than solving an equation?*

  Listen for evidence that students recognize that when multiplying or dividing by a negative number, the inequality symbol is reversed.

AT A GLANCE

Students revisit the problem on page 160 and connect the graph of the solution to the inequality modeling the problem.

STEP BY STEP

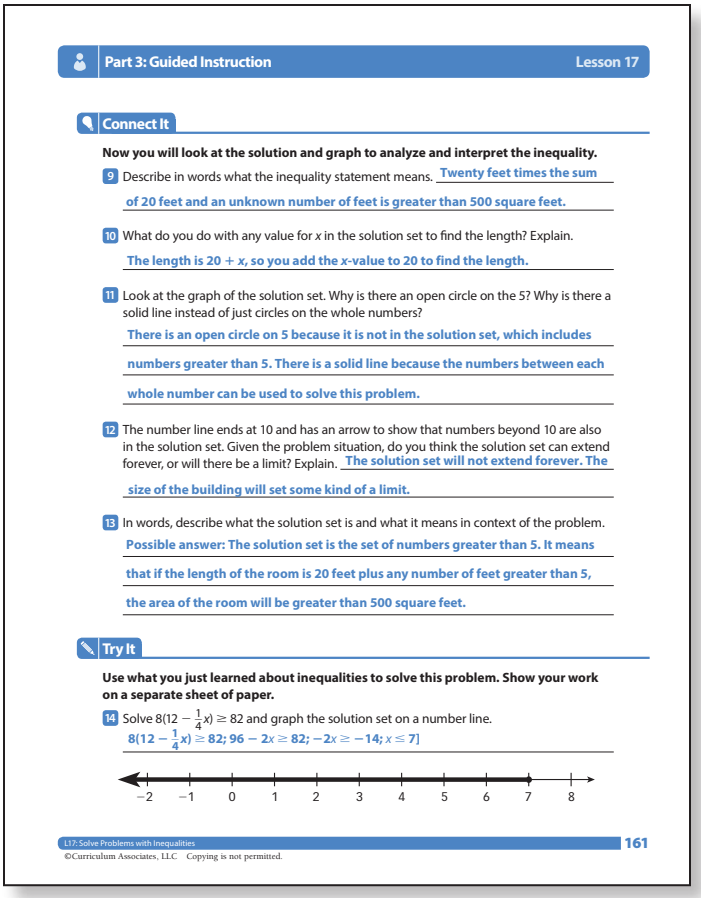
- Read Connect It as a class. Be sure to point out that the problems refer to the situation on page 160.
- After answering problems 9 and 10, focus on problems 11 and 12.
- Discuss with students whether it is reasonable in real life for a solution to the problem to include *all* fraction and decimal parts. [Students may say that some measurements are limited by the tools they have.]
- Ask, *How far can the line extend beyond 10?* [to infinity] Would that be a reasonable real-life solution? [no] Point out that mathematical solutions do not always reflect real-life solutions.
- Have several students answer problem 13 in their own words for the class.
- Have students do the Try It problem. Remind them to reverse the inequality symbol when multiplying or dividing by a negative number. Also explain that all numbers, not just integers, are included in a solution set unless a problem needs specific limits to make sense.
- After they have completed Try It, have students meet in pairs or small groups to discuss their answers. Encourage students to offer suggestions for changes if something is not correct.

SMP Tip: Students interpret the solution in light of the problem context and ask themselves whether the solution makes sense (*SMP 4*).

Concept Extension

Create context problems for inequalities.

- Have students suggest a word problem that would lead to the inequality in problem 14.
- Discuss with students different phrases used to suggest the inequality in their word problems.


Part 3: Guided Instruction Lesson 17

Connect It

Now you will look at the solution and graph to analyze and interpret the inequality.

9 Describe in words what the inequality statement means. Twenty feet times the sum of 20 feet and an unknown number of feet is greater than 500 square feet.

10 What do you do with any value for x in the solution set to find the length? Explain. The length is $20 + x$, so you add the x-value to 20 to find the length.

11 Look at the graph of the solution set. Why is there an open circle on the 5? Why is there a solid line instead of just circles on the whole numbers? There is an open circle on 5 because it is not in the solution set, which includes numbers greater than 5. There is a solid line because the numbers between each whole number can be used to solve this problem.

12 The number line ends at 10 and has an arrow to show that numbers beyond 10 are also in the solution set. Given the problem situation, do you think the solution set can extend forever, or will there be a limit? Explain. The solution set will not extend forever. The size of the building will set some kind of a limit.

13 In words, describe what the solution set is and what it means in context of the problem. Possible answer: The solution set is the set of numbers greater than 5. It means that if the length of the room is 20 feet plus any number of feet greater than 5, the area of the room will be greater than 500 square feet.

Try It

Use what you just learned about inequalities to solve this problem. Show your work on a separate sheet of paper.

14 Solve $8(12 - \frac{1}{4}x) \geq 82$ and graph the solution set on a number line. $8(12 - \frac{1}{4}x) \geq 82$; $96 - 2x \geq 82$; $-2x \geq -14$; $x \leq 7$]

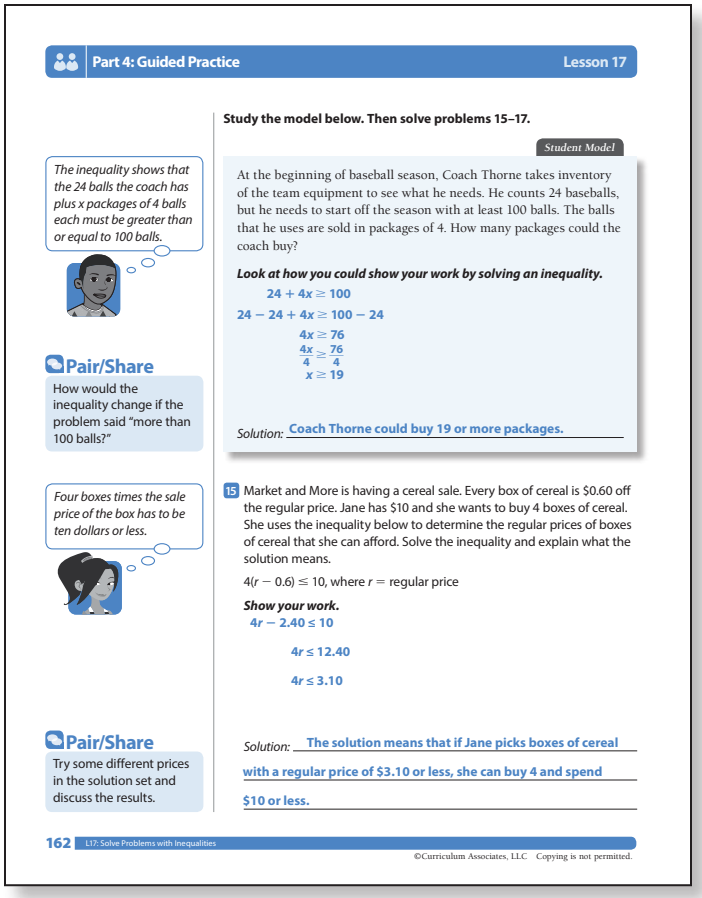
−2 −1 0 1 2 3 4 5 6 7 8

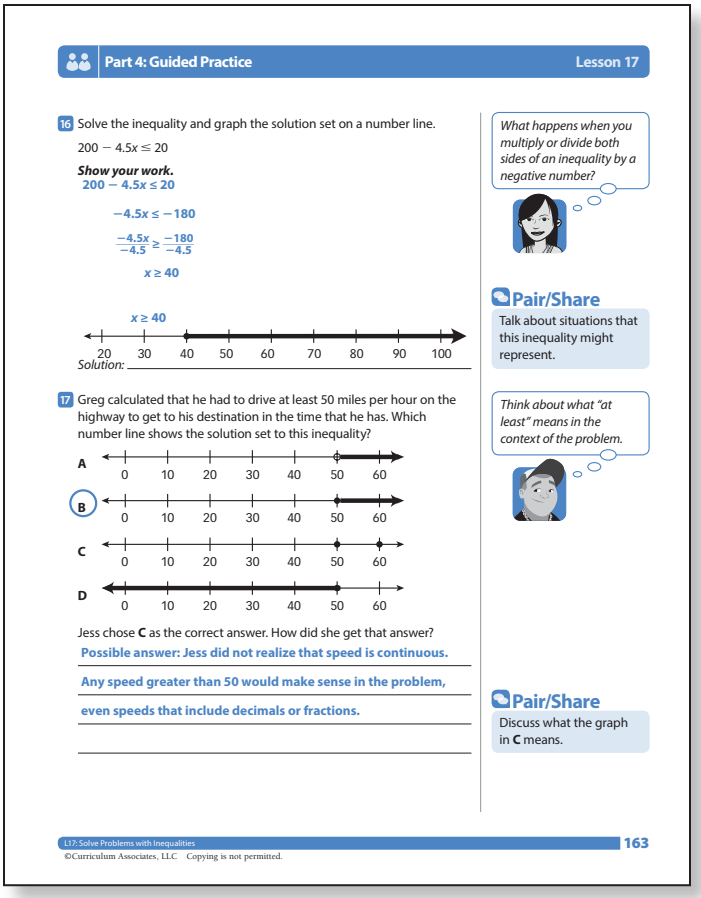
L17: Solve Problems with Inequalities 161

TRY IT SOLUTION

14 *Solution*: See answer on student page above; Students may solve the inequality by using the distributive property, subtracting 96 from both sides of the inequality, and then dividing both sides by −2 while reversing the inequality symbol.

ERROR ALERT: Students who wrote and graphed $x \geq 7$ did not reverse the inequality symbol.





AT A GLANCE

Students write inequalities to model word problems. They then solve the inequalities to find the answers or graph the solutions.

STEP BY STEP

- Ask students to solve the problems individually by writing, solving, and graphing inequalities.
- When students have completed each problem, have them Pair/Share to discuss their solutions with a partner or in a group.

SOLUTIONS

Ex Writing and solving an inequality is one way to solve the problem. Students may also use arithmetic and guess and check to solve the problem.

15 *Solution:* Jane can afford 4 boxes of cereal with a regular price of $3.10 or less; See possible student work above.

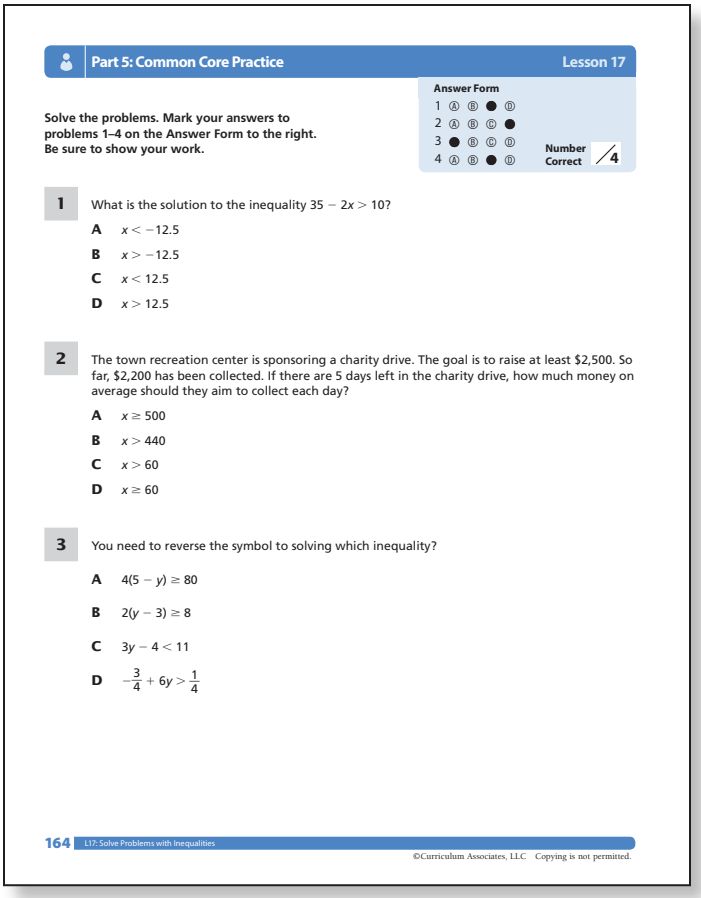
16 *Solution*: $x \geq 40$; See possible student work above, including graph.

17 *Solution*: **B**; Jess did not consider that speed can include decimals or fractions.

Explain to students why the other two answer choices are not correct:

A is not correct because it does not include 50. The problem says "at least," which means 50 is included.

D is not correct because it shows numbers less than or equal to 50.



Part 5: Common Core Practice Lesson 17

Solve the problems. Mark your answers to problems 1–4 on the Answer Form to the right. Be sure to show your work.

Answer Form
1 Ⓐ Ⓑ ● Ⓓ
2 Ⓐ Ⓑ Ⓒ ●
3 ● Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ ● Ⓓ
Number Correct ___/4

1 What is the solution to the inequality $35 - 2x > 10$?

A $x < -12.5$
B $x > -12.5$
C $x < 12.5$
D $x > 12.5$

2 The town recreation center is sponsoring a charity drive. The goal is to raise at least $2,500. So far, $2,200 has been collected. If there are 5 days left in the charity drive, how much money on average should they aim to collect each day?

A $x \geq 500$
B $x > 440$
C $x > 60$
D $x \geq 60$

3 You need to reverse the symbol to solving which inequality?

A $4(5 - y) \geq 80$
B $2(y - 3) \geq 8$
C $3y - 4 < 11$
D $-\frac{3}{4} + 6y > \frac{1}{4}$

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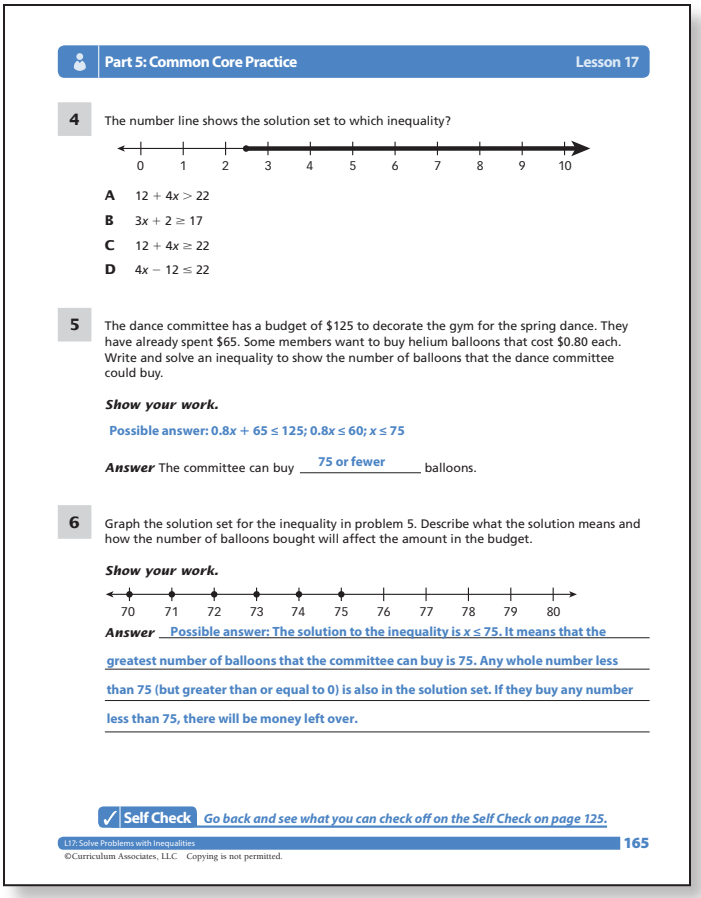

Part 5: Common Core Practice Lesson 17

4 The number line shows the solution set to which inequality?

0 1 2 3 4 5 6 7 8 9 10

A $12 + 4x > 22$
B $3x + 2 \geq 17$
C $12 + 4x \geq 22$
D $4x - 12 \leq 22$

5 The dance committee has a budget of $125 to decorate the gym for the spring dance. They have already spent $65. Some members want to buy helium balloons that cost $0.80 each. Write and solve an inequality to show the number of balloons that the dance committee could buy.

Show your work.

Possible answer: $0.8x + 65 \leq 125$; $0.8x \leq 60$; $x \leq 75$

Answer The committee can buy 75 or fewer balloons.

6 Graph the solution set for the inequality in problem 5. Describe what the solution means and how the number of balloons bought will affect the amount in the budget.

Show your work.

70 71 72 73 74 75 76 77 78 79 80

Answer Possible answer: The solution to the inequality is $x \leq 75$. It means that the greatest number of balloons that the committee can buy is 75. Any whole number less than 75 (but greater than or equal to 0) is also in the solution set. If they buy any number less than 75, there will be money left over.

✓ Self Check *Go back and see what you can check off on the Self Check on page 125.*

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AT A GLANCE

Students write and solve inequalities for word problems that might appear on a mathematics test.

STEP BY STEP

- First, tell students that they will write and solve inequalities for word problems. Then have students read the directions and answer the questions independently. Remind students to fill in the correct answer choices on the Answer Form.
- Explain to students that number lines can start at any number and do not have to include 0.
- After students have completed the Common Core Practice problems, review and discuss correct answers. Have students record the number of correct answers in the box provided.

SOLUTIONS

1 *Solution*: **C**; The inequality was reversed when both sides of the inequality were divided by −2.

2 *Solution*: **D**; Write $5x + 2{,}200 \geq 2{,}500$, subtract 2,200 from both sides, and divide both sides by 5.

3 *Solution*: **A**; Find the inequality that has a variable with a negative coefficient.

4 *Solution*: **C**; Solve each inequality and find the one with the solution $x \geq 2\frac{1}{2}$.

5 *Solution*: 75 or fewer; write the inequality $0.8x + 65 \leq 125$ to model the problem and then solve. See possible student work above.

6 *Solution*: See answer on the student page above. The committee can buy 75 or fewer balloons. The graph shows only whole numbers less than or equal to 75.

Assessment and Remediation

- Ask students to write and solve an inequality to solve this problem: *Denise has $10 and wants to buy pens that cost $0.80 each. How many pens can Denise buy and have at least $6 left?* [$10 - 0.8x \geq 6$; 5 pens or fewer]
- For students who are struggling, use the chart below to guide remediation.
- After providing remediation, check students' understanding. Ask students to write and solve an inequality to solve this problem: *Josh has $40 and wants to buy tubes of paint that cost $2 each. How many tubes of paint can he buy and still have at least $23 left over for canvas?* [$40 - 2x \geq 23$; $x \leq 8.5$, 8 tubes or fewer]
- If a student is still having difficulty, use ***Ready Instruction, Level 6,*** Lesson 20.

If the error is . . .	Students may . . .	To remediate . . .
5 or more pens	not have reversed the inequality symbol when dividing by −0.8.	Remind students that when multiplying or dividing both sides of an inequality by a negative number, they must reverse the inequality symbol.
5 or more pens	have solved the inequality $10 - 0.8x \leq 6$.	Remind students that "at least" means the same as "greater than or equal to."
0, 1, 2, 3, or 4 pens	have solved the inequality $10 - 0.8x > 6$.	Ask students to consider if "at least" includes $6 or not.
50 or fewer pens	have incorrectly divided −4 by −0.8 to get 50 instead of 5.	Remind students to check that their answers are reasonable.

Hands-On Activity

Find solutions to an inequality.

Materials: strips of paper, pencils

- Distribute strips of paper and have students make representations for money. Tell each student to make one $20 bill, two $10 bills, one $5 bill, four $1 bills, and 4 quarters, so each student has $50.
- Tell each student that they will spend $32 on a digital music player and can spend $1.25 per song to download.
- Ask students to find how many songs they can purchase using their "money."
- Finally, have students write and solve an inequality modeling the situation. Have them compare their answers. [$32 + 1.25x \leq 50$; 14 or fewer songs]

Challenge Activity

Write and solve an inequality involving average (mean).

- Tell students the following situation: *The high temperatures for two consecutive days were 83°F and 79°F. If the average (mean) high temperature, including a third day, was greater than 82°F, then what was the high temperature on the third day?*
- Have students write an inequality describing the problem and find the solution.
  [$\frac{83 + 79 + x}{3} > 82$; greater than 84°F]